

# A Review of Fluid Sloshing in Cylindrical and Rectangular containers under varied motions.

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**Abstract**— This paper illustrates detailed study of various methods and solutions that effects of liquid sloshing. The study includes the prismatic and cylindrical containers while in motion and in stationary. This gives us a picture about how sloshing can affect depending upon various parameters like velocity and acceleration of container, velocity of fluid inside moving container, viscosity of fluid, length of container and specific density of the fluid. The sloshing can be measured by using different approaches depending on the shape and motion of the container. The peak values of horizontal force and moment i.e., 0.4053 and 0.2038 remains unchanged when the Reynolds number is between 107 to 105. The level of liquid in the container plays a significant role in the dynamics of fluid sloshing, the peak values of horizontal force and moment at 0.4671 and 0.2388 respectively are reduced to 0.3060 and 0.1530 when fluid level in the container is increased from 0.3 m to 1.0 m. This is achieved using various techniques; one of them includes the use of vertical Baffles in liquid storage containers.

**Index Terms**— Analysis, Baffles, Mathematical modelling, Seismic motion, Simulation, Sloshing, Transportation.

## 1 INTRODUCTION

Sloshing is the oscillatory movement of any fluid inside the container. This oscillation is the resultant of various forces acting on the fluid, upon exposing it to irregular motion. Sloshing depends on the type of disturbance the container is experiencing, the shape of the container, fluid properties and various other factors. It is an important phenomenon, need to be taken under consideration while dealing with moving or stationary containers containing any kind of fluid [4]. Sloshing can be observed in fuel tanks of moving vehicles, cargo containers carrying fluids in bulk, marine and space vehicles, also seismically excited stationery storage containers, dams, reactors, and nuclear vessels. This fluid motion phenomenon is the direct cause of instability in the containers and hence may cause system failure. In moving vehicles, this instability becomes the cause of discomfort for the passengers [4]. When it is comes to the space vehicles, even the smallest value matters, therefore sloshing plays a huge role in the stability and working of the vehicle both in the earth's atmosphere and in space. Many research studies have been undertaken to precisely measure and find ways to prevent or reduce this sloshing behavior of the fluids under motion. In a research study by G Popov and S Sankar [4], using steady state solution derived from hydrostatic equations, the behavior of the liquid sloshing in a moving vehicle is observed. The sloshing was measured in terms of amplitude and damping frequencies. The observations showed that the linear steady state solution can be successfully applied on sloshing of smaller intensity. When the intensity is high non-linear model is more applicable for measurement of sloshing.

To In a research study done by Y.S. Choun, the sloshing response of large stationary storage tanks when subjected to seismic motions different peaks of acceleration to velocities ratio are observed. The Linear and nonlinear sloshing analysis are executed to identify the effects on sloshing in accordance with different inputs. The peak acceleration to velocity ratio is found out to be an excellent parameter with regards to sloshing measurement irrespective of the type of tank design.

Notations:

A: non-dimensional area occupied by the liquid, ratio to D<sup>2</sup>

A: dimensional area occupied by the liquid

D: reference length and container diameter

e: non-dimensional deflection of the free surface parabola

f: fill level, ratio to D

FHss: dimensional horizontal force in transient and in steady state

FVss: dimensional vertical force in steady state

g: gravity acceleration

G<sub>x</sub>: unit body force in X direction, ratio to g

G<sub>y</sub>: unit body force in Y direction, ratio to g

G<sub>n</sub>: unit body force in X direction in the middle

Hss: dimensional height of the free surface in steady state

Mss: dimensional overturning moment in transient and in steady state

Mss: non-dimensional overturning moment in transient and in steady state, ratio to 'liquid weight times D'.

P: non-dimensional pressure in transient, ratio to P<sub>0</sub>

Pss: non-dimensional pressure in steady state, ratio to pgD

δ: non-dimensional radius of turn, ratio to D

ρ: Density of the fluid in the container

## 2 SLOSHING IN MOVING CONTAINERS

The production and transportation of fuels and other fluids is of great importance for any nation. Fuels being transported through a network of pipelines and then further with tankers and containers of different shapes and geometry. On road tankers, filling is not complete, to maintain specific pressures

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$$U_x(t) = V(t)$$

$$U_y(t) = (-V^2(t))/R$$

$$\Omega(t) = (-V(t))/R$$

where the radius of curvature is,  
 $R = \xi_{xx} / (1 + (\xi_x)^2)^{3/2}$

Then, the basic modes of excitation can be obtained.

$$\vec{U} = U_x \hat{i} + U_y \hat{j}$$

To be more specifically, we shall assume a periodical motion profile for a study of harmonic excitation.

$$\xi(X) = (\theta_0/k) \cos kX$$

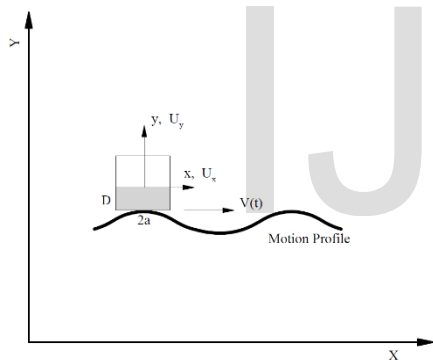
Where  $\theta_0$  is the elevation amplitude of the profile,  $K$  is the wave number of the profile and  $\lambda$  is the wavelength of the profile.

The velocity is described by:

$$\vec{X}(t) = V_0(1 + \delta \theta_0^2 \cos kV_0 t)$$

where  $V_0$  is a characteristic tank speed and  $\delta$  is a parameter of which characterized the response to the grade change.

Figure 3: The Motion Profile of the Tank.



**Rolling Motion:**

For a rolling motion about an axis on  $(X, Y) = (0, -d)$

$U_x, U_y$  and  $\Omega$  are specified as:

$$U_x = d\dot{\Omega}$$

$$U_y = -d\Omega^2$$

$$\Omega = \dot{\theta}$$

Where  $\theta$  is the angular displacement

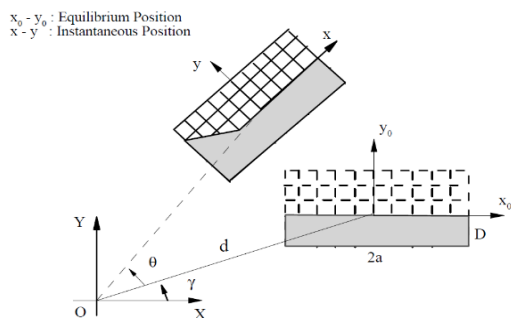


Figure 4: Rotating tank.

where all variables are now defined in the tank-fixed coordinate

system. The momentum equations yield the required expressions for two-dimensional flow in a rotating tank are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{\partial p}{\partial x} = -g\sin\theta - 2\theta'v - \theta''(y+d\sin\gamma) - U'x + \theta'^2(x+d\cos\gamma) + v(\nabla^2u)$$

$$(\frac{\partial v}{\partial t}) + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{\partial p}{\partial y} = g\cos\theta + 2\theta'u + \theta''(x+d\cos\gamma) - U'y + \theta'^2(y+d\sin\gamma) + v(\nabla^2v)$$

$\gamma$  is the equilibrium angle of the tank relative to the axis of rotation

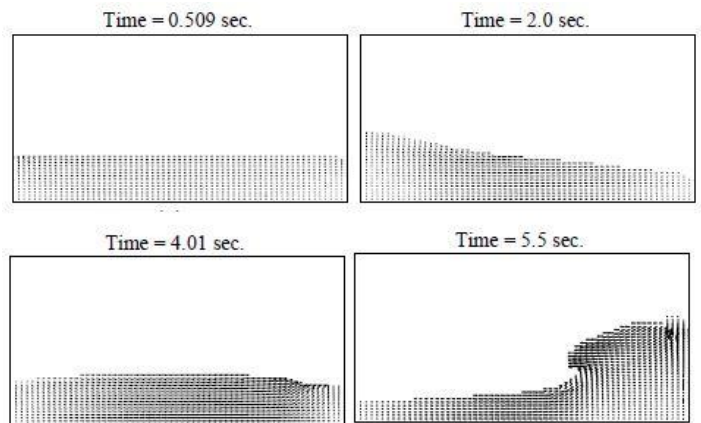
**2.3 Theoretical Analysis**

For a rectangular tank without any internal obstacles under combined external excitations (example. sway plus roll or surge and pitch), analytical solutions can be derived from the fundamental equations of fluid mechanics. These solutions can be used to predict liquid motions inside the tank, the resultant dynamic pressures on tank walls, and the effect of phase relationship between the excitations on sloshing loads.

The case is considered as a two-dimensional, rigid, rectangular tank without internal obstacles that is filled with inviscid, incompressible liquid. It is forced to oscillate harmonically with a horizontal velocity  $U_x$ , vertical velocity  $U_y$ , and  $\Omega a$  rotational velocity. Since the fluid is incompressible, the velocity potential must satisfy the Laplace equation with the boundary conditions on the tank walls [5].

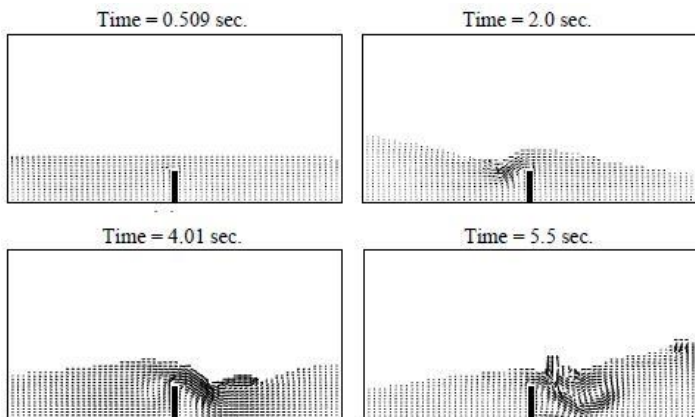
For the numerical solutions with the moving rectangular tank along a vertical curve, the  $\beta$  value ( $\beta = D^2/a^2$ ) of 0.0625 is used. For fill depth  $D = 4$  ft, the effective tank length ( $2a$ ) corresponding to  $\beta$  value is 32 ft. The computations, the excitation frequency of the tank  $\omega$  is varied from 0.1 to 1.3 rad/sec. and the corresponding wavelength of the periodical motion profile is taken as 1.43 ft.

Tank roll motion is defined by  $\theta = \theta_0 \sin\omega t$ , where  $\theta_0$  is the rolling amplitude. A typical numerical simulation of sloshing in a rigid rectangular tank with and without baffle is shown:



The liquid is responding violently causing the numerical solu-

tion to become unstable when the amplitude of excitation is increased. The instability may be related to the fluid motion such as the occurrence of turbulence, the transition from homogeneous flow to a two-phase flow and the introduction of secondary flow along the third dimension. The increased fill depth, the rolling amplitude and frequency of the tank with/without baffle configuration directly affected the degree of non-linearity of the sloshing phenomena. As a result of this, the phase shifting in forces and moments occurred [5].



It can be concluded from the baffled results that, for the lower excitation frequency, sloshing force and moment reduced slightly compared to those of unbaffled case.

### 3 SLOSHING IN STATIONARY CONTAINERS DUE TO SEISMIC MOTIONS

Liquid sloshing due to partially filled liquid storage containers is a realized physical phenomenon in today's world for example, fluid sloshing in storage tanks by earthquakes. This may lead to enormous damage to not only the economy but also the environment when considering a large of number of storage tanks if the excitation frequency is close to the natural frequency of the container [7]. The seismic tremor actuated fluid sloshing in a tank has been of incredible worry for as long as five decades and various broad examinations have been performed on the sloshing conduct under earthquake ground excitation. All things considered, numerous fluid tanks have been seriously harmed in past major earthquakes (Cooper 1997; Eshghi and Razzaghi 2005; Gates 1980; Hanson 1973; Haroun, Mourad and Izzeddine 1991; Hatayama 2008; Jennings 1971; Manos 1991; Manos and Clough 1985; Nielsen and Kiremidjian 1986; Rai 2003) [6].

Scientific solutions for the related problems are restricted to customary geometric tank shapes, for example, cylindrical, and rectangular. The idea of sloshing elements in cylindrical tanks is easily comprehended than that of rectangular tanks. In any case, scientific methods for anticipating large-amplitude sloshing are yet not completely developed [8].

#### 3.1 Sloshing in Rectangular tanks

Post-earthquake observations reports have demonstrated that fluid sloshing is one of the significant reasons for genuine harm in a tank and nature during seismic tremors. For open tanks with a lacking free-board, fluid materials, for example, oil and synthetic liquids may flood into the surrounding area

and result in the contamination of soil there. Because of roof-top tanks, an enormous sloshing wave will affect the divider or top of the tanks and may cause broad harm or failure of the tanks [6]. Many different scientists have approached this concern with different methods which are as follows:

#### 3.2 The peak ground acceleration to velocity (A/V) ratio

The ground movements, showing an enormous movement, for the most part bring about high A/V ratio and very large spectral acceleration values at short periods, though the ground movements, containing intense, long duration acceleration pulses, will generally lead to low A/V ratios and pronounced spectral acceleration values for a moderate or long period [10]. The peak ground acceleration (PGA) and peak ground velocity (PGV) are usually caused by seismic waves of different frequencies. The PGA is associated with high frequency waves, whereas the PGV is related to moderate or low-frequency waves. As a result, one would expect that ground motions near an earthquake source have higher A/V ratios than ground motions at a long distance from the source of the seismic energy release [11]. The earthquake records were obtained from 9 different earthquake events with magnitudes ranging from 6.1 to 7.6. The selected records have distance from epicenter ranging from 7.1 to 69.1 km, original PGAs ranging from 0.084 to 1.585 g, and peak A/V ratios from 0.16 to 3.81 g/m/s [6].

#### 3.3 Two phase sloshing analysis with Volume of fluids (VOF) method

The geometry used in this method was of a two-dimensional rectangular domain where the free surface is limited by the vertical walls [9].

The following are the assumptions made for this analysis:

- A. Fluid is incompressible.
- B. Fluid is inviscid.

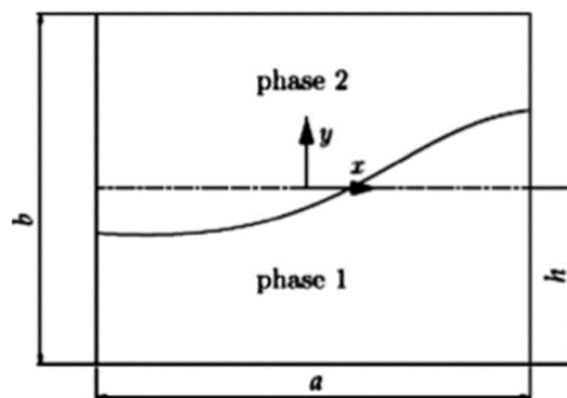


Figure 5: Geometry of a 2 Phased rectangular container.

The use of VOF method for sloshing problem implies then study of 2 fluids with densities 1000 kg/m<sup>3</sup> and 1.3,50,200,800 kg/m<sup>3</sup> (of bottom and top fluids respectively) that must completely fill the tank with dimensions a(width)= 1.0m and



b(height)=1.0m and the initial free surface is at height  $h = 0.5m$  from the bottom of the tank surface and lateral excitation of fluid given by  $x=0.05\sin(3t)$ . Mesh convergence process for the same conditions were obtained by the STARCCM+ software. Figure below appreciates the mesh convergence process for the same.

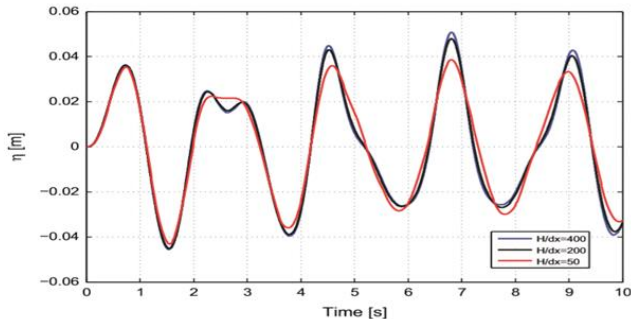


Figure 6: Convergence analysis.

### 3.4 Sloshing in Cylindrical tanks

Sloshing in Cylindrical containers is generally described by three-dimensional flow. Estimation of forces, moments and natural frequency are few of the fundamental problems in liquid sloshing. Even though sloshing is a troublesome scientific issue to understand systematically, early treatments of this issue were done with scientific techniques built up on potential flow theory ignoring all viscous effects. Approximately, Jacobsen gave the first solution for a rigid cylindrical container under horizontal motion based on a closed form solution of the Laplace equation which satisfies specified boundary conditions [9].

Horizontal and Vertical forces on the container: -  
 Total Horizontal Fluid forces =  $-(\alpha J(\lambda a) \rho) / \lambda \tanh \lambda h$   
 Total Vertical Fluid Forces =  $+0.202 \rho / \cosh^2(\lambda h) \alpha^2$

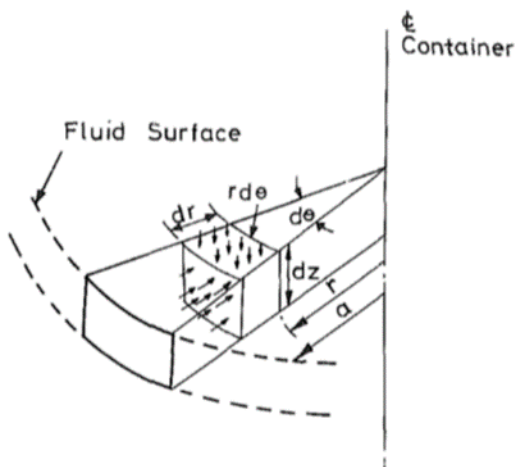


Figure 7: Element of fluid with boundary conditions

### Amplitude Equations

The pressure equation for unsteady incompressible fluid flow and the equation for the free surface kinematic condition of fluid particles are found [13]. And after that when we put the boundary conditions at the free surface, pressure  $P = 0$  and the free surface elevation  $n=z$  in the pressure equation then we

got:

The first fluid amplitude equation is:

$$\alpha - (g + Z_0) a_{11} - X_0 (2a / (\epsilon^2 - 1)) (\epsilon) = 0$$

The second fluid amplitude equation is:

$$(-\partial\phi/\partial z) = (\partial n/\partial t) = (-\partial\phi/\partial r) (\partial n/\partial r) (-1/(r^2)) (\partial\phi/\partial\theta) (\partial n/\partial\theta) |_{z=n}$$

Then after doing all substitution, we get the second fluid amplitude equation which we got is

$$a_{11} + \alpha_{11} = 0$$

Where,  $J$  = Bessel function of the first kind

$\rho$  = fluid density

$h$  = fluid depth

$\phi$  = velocity potential function

$g$  = gravity

$\theta$  = angular motion about container.

## 4 Results and discussion

The steady solution for viscous fluids and vehicles under turning provides measurement and analytical solution of the pressure, the different forces, and the extent of motion of the liquid inside the containers. The steady state solution shows that the main parameters influencing the liquid motion the moving containers are Reynolds number, the acceleration provided to the container and the level of the liquid inside the container.

### 4.1 Effect of Viscosity

G Popov et. al [3] in an experimental showed the effects of different parameters on the sloshing. It depicts that when the Reynolds number is in the range 107 to 105 there is no significant effect on the frequencies and amplitude of the sloshing parameters. It is below 103 when the many differences are observed in amplitude and frequency. The experimental observations are presented in the table 1.

I Table

$Re$	Peak values		First damped natural frequency	Undamped theoretical frequency
	$F_H$	$M$		
$10^7$	0.4053	0.2038	1.151	1.201
$10^6$	0.4053	0.2038	1.151	
$10^5$	0.4053	0.2037	1.151	
$10^4$	0.4031	0.2027	1.143	
$10^3$	0.4008	0.2016	1.139	

Reynolds number being an inverse function of the viscosity of the liquid shows a clear view that the significant differences in frequency and amplitude observed are at the higher viscosity and remains unchanged at lower viscosity.

### 4.2 Effect of applied acceleration

The acceleration has a significant effect on the sloshing of the fluid, as depicted by the experimental data. When the acceleration is increased keeping other parameters constant, it is observed that the horizontal forces and moment are increases. On the other hand, the frequency of the slosh decreases after a certain increment in the acceleration [9].

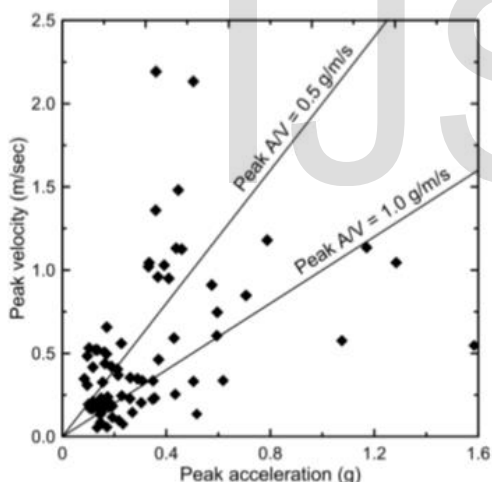
**II Table**

$G_x$	Peak values		Damped Frequency
	$F_H$	M	
0.1	0.1412	0.0871	1.198
0.2	0.2811	0.1510	1.180
0.3	0.4053	0.2037	1.151
0.4	0.5061	0.2613	1.004
0.5	0.5952	0.3082	0.883

**III Table**

f	Peak values		Damped Frequency
	$F_H$	M	
0.3	0.4671	0.2388	1.037
0.5	0.4229	0.2140	1.098
0.6	0.4053	0.2037	1.151
0.8	0.3720	0.1881	1.221
0.9	0.3282	0.1652	1.555
1.0	0.3060	0.1530	-

In the studies done on sloshing in stationary containers under seismic acceleration shows effect of it for different input motions with different ground motion Acceleration to Velocity (A/V) ratios. The study done by Y.S. Choun shows that regardless of the shape of storage tank, the peak A/V ratio were divided in 3 categories as  $A/V \leq 0.5 \text{ g/m/s}$ ,  $0.5 \text{ g/m/s} < A/V \leq 1.0 \text{ g/m/s}$ , and  $A/V > 1.0 \text{ g/m/s}$ . These categories showed that there is tremendous sloshing in broader tanks under a strong seismic motion and the sloshing effect decreases with increase in A/V ratio [9].



**Figure 8:** Peak acceleration versus peak velocity for different A/V ratios.

**4.3 Effect of level of fluid**

Experiment shows, the level of the liquid inside the container is an important parameter of the sloshing. Table 3 shows how the increase in fill level from 0.3 to 1.0 influences the horizontal force and moment to slowly decrease in magnitude. Also, it is noticed that the frequency is significantly increases on increases the fluid level in the container [4].

**5 CONCLUSIONS**

This review paper presents the brief discussion on the various novel studies conducted on the Fluid sloshing to analyze its behavior and effects. In general, we have studied the liquid sloshing inside various shapes of partially filled tanks in motion and stationary. The steady state solution applied in the case the moving containers shows that only after a certain increase in viscosity or at larger values of viscosity, significant changes in the sloshing parameter like horizontal force, moment and frequency are observed. The vehicle acceleration and deceleration also influence the fluid sloshing of the fuels in the tanks. When the acceleration is increased the horizontal force and moment increases. However, the frequency decreases. Analytic solution for liquid excitation has been observed theoretically and experimentally. It has been observed that liquid reacts violently causing the numerical study to fail under higher amplitudes. This instability leads to the occurrence of turbulence in the container. When we increase the fluid level in the container the rolling amplitude and frequency of the tank effect the container by increasing the degree of liquid sloshing motion. As a result of this the phase shifting has occurred and caused moments to change. The horizontal forces and moments showed a decrement when fluid level increased. Contrary to this decrement the frequency of the slosh increased gradually.

The sloshing behavior in a liquid tank depends on the Peak ground velocity, as well as on the Peak ground acceleration of seismic motions. The peak A/V ratio is a significant seismic hazard parameter in addition to the peak ground acceleration in the seismic design of liquid storage tanks but, the current seismic design codes and standards on liquid-containing tanks do not consider the peak A/V ratio to determine the seismic design forces. Therefore, the peak A/V ratio of earthquake ground motions should be included in the seismic analysis and design procedures of liquid storage tanks. The study of sloshing phenomena as done in previous section considering long period and long duration ground motions provides a

simple and efficient methodology for predicting the dynamic response of liquid tanks.

Reduction of sloshing and its effect can provide better and safer environment. To prevent sloshing, obstacles are placed inside the containers to restrict the sloshing. These obstacles are called baffles. Fluid separates around baffles inside the container, the energy of the oscillating liquid is dissipated which results in reduction of amplitude and corresponding sloshing forces on the walls. Baffles are regular devices in lessening sloshing impacts in moving containers. A few examinations have been done in such manner, particularly on their application in fuel containers of space vehicles whose stability is sensitive to uncontrolled motions. The impacts of a baffle on fluid motions can be separated into changes in frequency and damping proportion of sloshing mode. In circular-cylindrical shaped tanks, the frequency can be up to 15% higher than the un-baffled tank value when an even ring baffle crosses the surface (halfway by a clear decrease in the free surface breadth) and up to 10% lower than the un-baffled tank value when the free surface is around one baffle width above the baffle. The progressions in damping proportion of sloshing mode can be effective in reducing the sloshing impacts, for example, hydrodynamic weight and sloshing sufficiency particularly with an excitation frequency equivalent or near the reverberation frequency. Experiments show that the damping ratio increases with increase of relative sloshing height and baffle height (from the bottom of tank) for both kinds of baffles, provided the baffle is not uncovered during the sloshing, because the theoretical models were subject to this assumption.

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